Prior to Replacement

by A. Della Corte

Mathematicians, especially those working in pure mathematics, form a singularly misunderstood community among those engaged in hard sciences. Their professional activity and social dynamics reach the outside world only in a heavily distorted form. Films and novels about great mathematicians select figures that are exceptional not only for the level of their contributions but, above all, for their human peculiarities. Many popular accounts likewise offer a romanticized version of the mathematician's work, and even the history of science abounds in apocryphal anecdotes that emphasize the moment of ingenious discovery (*Eureka*!) while overlooking other necessary and no less profound components of research. Nor does the school system (which has enough urgent daily concerns on its own) typically succeed in conveying a realistic sense of what research in this field truly entails.

In reality, the community of pure mathematicians consists, for the overwhelming majority, of small "tribes," each scattered across many countries, gathering periodically at conferences and workshops. Each tribe works on a set of topics that is minuscule when compared with the vast universe of mathematics as a whole, producing results at a pace generally unhurried when measured against the prolific output of theoretical physicists, computer scientists or applied mathematicians. And yet, despite their distinctness, these tribes share methods, criteria, and language at a level of universality that few other sciences attain. Thematic localism, indifference to geographical borders, and methodological universalism coexist in pure mathematics in a balance largely free of internal tensions.

Only in rare cases do mathematicians confront problems that are really famous, those four or five whose names an educated layperson may have read or heard about. In almost all cases, instead, they study problems regarded as significant by a few dozen or a few hundred people worldwide, guided primarily (though not exclusively) by reasons that are internal to a given research tradition. Indeed, one of the essential functions of these micro-communities is to reshape the general direction of inquiry, to cultivate new branches, and sometimes to prune those that have become sterile. To perceive, as is often said, the *inevitability* of the larger picture that gradually emerges from the details.

In practice, research typically consists of a wide collection of activities: exploring the literature, studying foundational papers in depth, discussing ideas with students, arranging a call with colleague X specialist in technique Y. Only a proportionally small part of one's time is devoted to the brute effort of attempting to "break through" a difficulty, after the problem has been selected, clarified, delimited, and stripped of inessential components. That effort is itself, often, collective, carried out before a blackboard, in daily work that most often unfolds without particular haste, without promotional excess, soberly anchored to minute details.

Here is, instead, what never happens in the real world: the "boss" (who perhaps understands nothing of mathematics) enters a mathematician's office on Monday morning, points to a statement, and commands "prove this by Friday." That is simply *not* how it works. The raw solution to a problem, the technical result consisting of the rigorous proof of a proposition, the detailed construction of an example demonstrating that the mathematical phenomenon X is possible, almost never possesses immediate market value. Society has not, for centuries, considered it useful to have a broad community of professional pure mathematicians solely for the prompt execution of immediately marketable micro-tasks.

Of course early-career researchers (undergraduates, doctoral students, postdoctoral researchers) sometimes carry out tasks under the direction of a supervisor. Even in such cases, though, the focus is as much on the process as on the result. One of the primary duties of the young mathematician is to learn techniques, absorb known results, and internalize ways of thinking before acquiring sensibility and independence. In summary, the social legitimacy of pure mathematics has never rested on surely predictable, short-term return.

Why, then, does the idea of replaceability spread so easily? Why, in particular, is the narrative so widespread that pure mathematicians are at risk of replacement by Large Language Models (LLMs) and artificial intelligence systems more broadly?

The widely publicized achievements of LLMs (such as results in Mathematical Olympiads, performance on Frontier-type benchmarks, the intriguing "First Proof" experiment) have all, in effect, involved the execution, more or less successful, of decontextualized tasks assigned by an external agent. One may reasonably question, with solid arguments, whether such execution is genuinely reliable for problems at the level encountered in actual research. A balanced analysis, such as that published in *Scientific American* ("AI just got its toughest math test yet. The results are mixed", by J. Howlett and C. Cameron) and shared in its social media channels by the Harvard Mathematics Department, may plausibly conclude that talk of replacement is premature. Even this defensive line, though, implicitly accepts a crucial misunderstanding, in that it portrays the pure mathematician as an executor of externally assigned tasks.

A persuasive narrative of replaceability requires, prior to replacement, this silent paradigm shift: the reduction of mathematics' intrinsic and social significance to its procedural and passive dimension. For this reason, it may be healthier to cultivate a quiet resistance on a deeper plane, beginning with a more realistic transmission to the outside world of what daily research work actually looks like and of the social dimensions of mathematical life. In particular, it seems important to foster greater awareness, both externally and internally, of the significant social role played by the community of pure mathematicians simply by valuing meticulous care, sobriety, rigor and blunt, compelled honesty about one's own mistakes: qualities that elsewhere are often penalized by prevailing social dynamics. Something of the romantic aura surrounding the mathematician might be lost in such a clarification, but much would be gained in terms of perceiving the community's place within the broader ecology of social life and its contribution to shared culture. Above all, beyond any effect on the brand of "pure mathematics," it would be a more earnest account. Closer to reality.

But why does the narrative of replacement gain such traction? Two powerful elements sustain it, independently yet synergistically.

The first is psychological. LLMs are designed and trained to resemble a human interlocutor as closely as possible. When asked to prove a mathematical statement, they produce a text, impeccable in terms of syntax and wording, in which the argument unfolds as it would in a paper, a book, or a lecture. This discursive character often does not assist human verification; indeed, it can make error detection more difficult. Phrasing, technical jargon, and notation appear so polished that one must remain vigilant against unpredictable and, from a human standpoint, unintelligible mistakes that may surface at any moment, mistakes quite different from the typical subtleties a human mathematician might miss. It is for this reason that there is strong interest within the scientific community in hybrid forms of AI in which LLMs are coupled with systems capable of translating reasoning into formally verifiable language. Progress in this direction is rapid and will likely have bigger consequences for mathematics than any improvement in LLMs considered alone.

Now imagine, for a moment, that LLMs did not exist in their present conversational form, and that instead we had only highly advanced, mathematics-specific AI tools whose output was not discursive text, but a procedural list. Say, something of this kind:

- Apply Thm 4.31 of paper X with $f = \sinh(x^2)$ + compactness assumption;
- Measure-theoretic pigeonhole principle $\longrightarrow \exists$ positive-measure element of weakly measurable [in the sense of book Y, p. 57] partition Σ;
- Perform integration over $D \backslash \partial U$;

… … …

Such tools would be of similar help (and perhaps even easier to use) to the "brute force" part of the work of professional mathematicians. The narrative of replacement, however, would be likely far less persuasive among non-professionals. The shared experience of "chatting" with an LLM, which subtly instills the impression of interacting with an individual, would be absent. It would also be more transparent that the effective use of such tools requires substantial expertise and cannot be judged by superficial features. LLMs thus provide a potent psychological foundation for the idea that not only components of certain activities, but entire categories of human beings are replaceable.

The second element is sociological. For decades, in the United States before and then in Europe, there has been a tendency to reduce cultural practices to instrumental functions. This tendency includes the gradual transformation of universities (and, in many countries, schools) into institutions modeled on private corporations, governed by corporate-like managerial criteria even where they formally remain public institutions, as they usually do, for instance, in Italy, France, Germany. This development has produced, inadvertently, a constructive interference with an anti-intellectual current that identifies cultural elites as one of its principal adversaries. This phenomenon intensified after COVID but long predates it, as argued in Richard Hofstadter's *Anti-Intellectualism in American Life* (1963) and, more recently, by Tom Nichols in *The Death of Expertise* (2017). Elements of the political and economic elite have, of course, sought to ride this wave. Thus, a culture of corporate efficiency and a form of populist neo-obscurantism, though born of distinct premises, have converged in casting doubt upon the survival of long, international, free intellectual traditions not reducible to market logic. Pure mathematics is by no means alone in this condition: classical philology, Egyptology, ethology, anthropology, paleontology, the history of prehistoric art and dozens of other disciplines not promising economic self-sufficiency through their own outputs are similarly exposed. In all these cases, the principal risk factor is not market pressure alone (which such disciplines have, as a matter of fact and until now, survived), but its combination with an ideology that regards sophisticated culture as a superfluous, dubious distortion of an otherwise simple, stable, reassuring reality.

LLMs and, more generally, GenAI tools offer, for the first time in history, a vaguely credible alternative to the demanding and often inhospitable high road that traditionally passed through cultural institutions and their laborious curricula. An alternative whose opaque nature and anthropomorphic character rhetorically fuse the stamp of technological efficacy with the intoxicating promise of subversive prodigy.

Among all disciplines, pure mathematics occupies a particular place in the public discourse because of the misplaced conviction that it is more directly tied than others to “higher” faculties of intelligence. The reductive image of mathematical research conveyed by the media tends to downplay its affinities with other fields, neglect its social specificities, and ignore the weight of its traditions, emphasizing instead only a dimension of individual “brute force”. Because of this supposed special intellectual status, the narrative of replaceability in this domain acquires a particularly penetrating force. And for this reason it is essential to remain lucid about what we inadvertently concede, again, prior to replacement, about the crawling paradigm shift from a rich and differentiated social activity to the execution of externally generated procedures. For a convincing account of the replaceability of pure mathematics would risk legitimizing a broader sociological model in which all forms of contribution to collective life appear, ultimately, structurally inessential beyond their integrated role as an efficient provider for certain specific kinds of outputs.

I am persuaded, overall, that pure mathematics, heir to a millennial tradition and survivor of immense historical catastrophes, will endure this transitional phase, adapting as it has always done. Its role within the social ecology, however, may change, and not necessarily for the better: the way its community inhabits the world, the degree to which its distinctive virtues (care for detail, freedom from haste, clarity, honesty) percolate into shared culture. Even this attenuated form of marginalization, rather than outright disappearance, would constitute a loss difficult to compensate, and perhaps one whose consequences would prove hard to circumscribe.